Hwk 7 MAT 5190 M. ALVO

on 13

7.34 The group is given by $g_1(x) = n - x$, $g_2(x) = x$. Under this group.
If $Y = n - X$, then
$Y = g_1(X) \sim B(n, 1-\theta) \quad \therefore \quad \bar{g} = 1 - \theta$

The invariant estimator T must satisfy

a) $T(x) = 1 - T(n-x)$

(5)

b) The Bayes estimator is

$$\hat{p}_B = \frac{y + \alpha}{\alpha + \beta + n}$$

For this to be invariant we must have

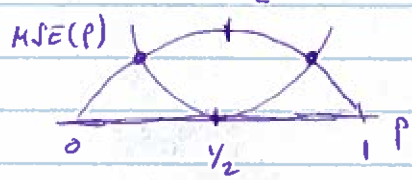
$$\frac{y + \alpha}{\alpha + \beta + n} = 1 - \frac{n - y + \alpha}{\alpha + \beta + n}$$

$$= \frac{y + \beta}{\alpha + \beta + n}$$

$\therefore \quad \alpha = \beta$

c) $$MSE = E\left[\hat{p}_B - p\right]^2 = E\left[\frac{y}{\alpha+\beta+n} - \frac{np}{\alpha+\beta+n}\right]^2 + \left[\frac{np+\alpha}{\alpha+\beta+n} - p\right]^2$$

$$\underset{(\alpha=\beta)}{=} \frac{np(1-p)}{(2\alpha+n)^2} + \left(\frac{\alpha(1-2p)}{2\alpha+n}\right)^2$$



Consider the rule which has constant MSE for all p. Then

at $p = \frac{1}{2}$, $MSE\left(\frac{1}{2}\right) = \dfrac{n}{4(2\alpha+n)^2}$

at $p = 0$, $MSE(0) = \dfrac{\alpha^2}{(2\alpha+n)^2}$

Equating we have that $\alpha = \sqrt{n}/2$

Hence, $\dfrac{y + \sqrt{n}/2}{\left[2\sqrt{\frac{n}{2}} + n\right]}$. Alternatively $\dfrac{\partial MSE(p)}{\partial p} = \dfrac{(1-2p)(n-4\alpha^2)}{(2\alpha+n)^2} = 0$

yields a minimum $\Rightarrow \alpha > \sqrt{n}/2$

7.39 $\int f(x;\theta)\,dx = 1$

Differentiate with respect to θ $\int \frac{\partial}{\partial\theta} f(x;\theta)\,dx = 0$

i.e. $\int \left[\frac{1}{f(x;\theta)} \cdot \frac{\partial f(x;\theta)}{\partial\theta}\right] \cdot f(x;\theta)\,dx = 0$

(2) $\int \left[\frac{\partial \log f}{\partial\theta}\right] \cdot f(x;\theta)\,dx = 0$

Differentiate again $\int \left[\frac{\partial^2 \log f}{\partial\theta^2}\right] f(x;\theta)\,dx + \int \frac{\partial \log f}{\partial\theta} \cdot \frac{\partial f(x;\theta)}{\partial\theta}\,dx = 0$

$$\int \frac{\partial^2 \log f}{\partial\theta^2} \cdot f\,dx + \int \left(\frac{\partial \log f}{\partial\theta}\right)^2 f\,dx = 0$$

7.41 a) $E\left[\sum a_i X_i\right] = \left[\sum a_i\right]\mu = \mu$ if $\sum a_i = 1$

b) $Var\left[\sum a_i X_i\right] = \sigma^2 \sum a_i^2$.

(2) $U = \sum a_i^2 + \lambda\left(\sum a_i - 1\right)$

$\frac{\partial U}{\partial a_i} = 2a_i + \lambda = 0 \Rightarrow a_i = -\lambda/2$ for all i

Hence all the a_i are equal to $\frac{1}{n}$

7.48 a) The Fisher information for the Binomial was shown in class to be $\frac{n}{p(1-p)}$

(4) $\therefore$ the CR lower bound is $\frac{p(1-p)}{n}$ which is attained by $\bar{X}_n$

b) We know by independence $E\prod X_i = p^4$; also $\sum X_i$ is complete and sufficient for p, Hence $E\left[\prod X_i \mid \sum X_i\right]$ is better by Rao Blackwell. Since $X_i = \begin{cases}1\\0\end{cases}$, $\phi(t) = E\left[\prod X_i \mid T = \sum x_i\right] = P(X_1 = \cdots = X_4 = 1 \mid T = t)$

$$= p^4\, P\left[\sum_5^n X_i = t-4\right] / P\left[\sum_1^n X_i = t\right] = \binom{n-4}{t-4} \Big/ \binom{n}{t}$$